Entry Task: Your company makes two household cleaners: Miracle Bathtub Cleaner and Speedex Floor Cleaner. Your daily production of both cleaners combined is limited to 2,000 gallons. Your daily sales of Miracle Bathtub Cleaner never exceed 1,200 gallons, and your daily sales of Speedex Floor Cleaner never exceed 1,400 gallons. Finally, you make $\$ 1.00$ profit on each gallon of Miracle Bathtub Cleaner that you sell and \$2.00 on each gallon of Speedex Floor Cleaner that you sell. Determine the amount of each cleaner you should produce in order to maximize profit.
STEP 1: "...amount of each cleaner..."

| $x=$ |
| ---: |
| $y=$ |


|  | x | y |
| :---: | :---: | :---: |
|  |  |  |

STEP 2: Constraints and Objective?
STEP 3: Graph
STEP 4: Corners

Closing Tues: 4.2
Closing Thurs: $\quad 5.1 / 5.2$ and 5.3 Exam 2 is Tuesday, Nov. $22^{\text {nd }}$. No lecture Wed, Nov. $23^{\text {rd }}$.

## 5.1/5.2 Exponentials \& Logarithms

 (Get your calculator out!)Def' $n$ : An exponential function can be written as

$$
\begin{aligned}
& f(x)=A b^{x} \\
& A=f(0)=\text { the } y \text {-intercept } \\
& b=\text { the base }
\end{aligned}
$$

The most used base in applications is $e=2.71828182 \ldots$... (Euler's constant)

For $0<b<1$, exponential decay Example: $f(x)=6(0.5)^{x}$


If $\mathrm{b}>1$, exponential growth Example: $f(x)=5(2)^{x}$


Exponential functions are everywhere you look in business!
Examples
Savings Accounts


## Economic Growth



## Skills Review <br> Power/Root/Exponent Facts

| Rule | Example | Example |
| :--- | :--- | :--- |
| $b^{0}=1$ | $3^{0}=$ | $7 e^{0}=$ |
| $b^{\left(\frac{1}{n}\right)}$ <br> $=\sqrt[n]{b}$ | $16^{1 / 2}=$ <br> $8^{1 / 3}=$ <br> $9^{3 / 2}=$ | $\sqrt[5]{e^{x}}=$ |
| $b^{-x}=\frac{1}{b^{x}}$ | $2^{-3}=$ <br> $27^{-1 / 3}=$ | $\frac{1}{e^{x}}=$ |
| $b^{x+y}$ <br> $=b^{x} b^{y}$ | $x^{2} x^{3}=$ | $e^{x} e^{3 x}=$ |
| $\frac{b^{x}}{b^{y}}=b^{x-y}$ | $\frac{x^{8}}{x^{3}}=$ | $\frac{e^{5 x}}{e^{2 x}}=$ |
| $\left(b^{x}\right)^{y}$ |  |  |
| $=b^{x y}$ | $\left(x^{2}\right)^{3}=$ | $\left(e^{3}\right)^{x}=$ |

## Solving with powers and roots

## Powers/Roots:

$$
y=x^{n} \leftrightarrow \mathrm{y}^{(1 / n)}=\sqrt[n]{y}=x
$$

(Taking an even root? you need " $\pm$ ")

Get out your calculator
Solve

1. $x^{2}=7$
2. $\sqrt{y}=3$
3. $t^{5}=60$
4. $\sqrt[5]{w}=3$
5. $\sqrt[3]{(2 x-1)^{5}-5}=3$

6. $2 e^{10 x}-5=7$

Solve

1. $\mathrm{e}^{\mathrm{x}}=7$
2. $\ln (y)=14$
3. $e^{3 x}=4$

$$
\text { 4. } 2 e-5=1
$$

## Exponentials/Natural Logarithm

$$
\begin{gathered}
y=e^{x} \leftrightarrow \ln (y)=x \\
y=b^{x} \leftrightarrow \log _{\mathrm{b}}(y)=x
\end{gathered}
$$

Let's find some new buttons on your
calculator (get your calculator out)
Example 1: Using your calculator:
STEP 1: Compute $\mathrm{e}^{2} \approx$ BLAH
STEP 2: Compute $\ln (B L A H)=$ ??
Example 2: Using your calculator:
STEP 1: Compute $\ln (3)=$ BLAH
STEP 2: Compute $\mathrm{e}^{\text {BLAH }}=$ ? ?

## Logarithm Facts

| Rule | Example |
| :--- | :--- |
| $1=e^{0}$ and $\ln (1)=0$ |  |
| $e=e^{1}$ and $\ln \left(e^{1}\right)=1$ |  |
| $\ln \left(e^{2}\right)=2, \ln \left(e^{3}\right)=3$, and so on... |  |
| $\ln (a b)=$ $\ln (3)+\ln (5)=$ <br> $\ln (a)+\ln (b)$  |  |
| $\ln \left(\frac{a}{b}\right)=$ $\ln (20)-\ln (2)=$ <br> $\ln (a)-\ln (b)$  |  |
| $\ln \left(a^{b}\right)$ <br> $=b \ln (a)$ | $\ln \left(2^{x}\right)=$ |
| $\ln \left(e^{x}\right)=x$  <br> $e^{\ln (y)}=y$ $\ln \left(e^{2}\right)=$ <br> $\mathrm{e}^{\ln (3)}=$  |  |

Example:
Compute $\ln \left(\frac{e^{3} e^{4}}{e^{2}}\right)$

## Review of all solving facts

We solve by using inverses in the correct order to get the variable by itself

| Equation | Inverse |
| :--- | :--- |
| $x+3=14$ | $x=$ |
| $y-5=22$ | $y=$ |
| $3 t=16$ | $t=$ |
| $\frac{m}{0.2}=100$ | $m=$ |
| $x^{2}=7$ | $x=$ |
| $\sqrt{y}=3$ | $y=$ |
| $t^{5}=20$ <br> $\sqrt[5]{w}=3$ | $t=$ |
| $e^{x}=10$ | $x=$ |
| $\ln (y)=3$ | $y=$ |
| $5^{t}=60$ | $t=$ |

